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QCD and deep inelastic scattering

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Slides available at:

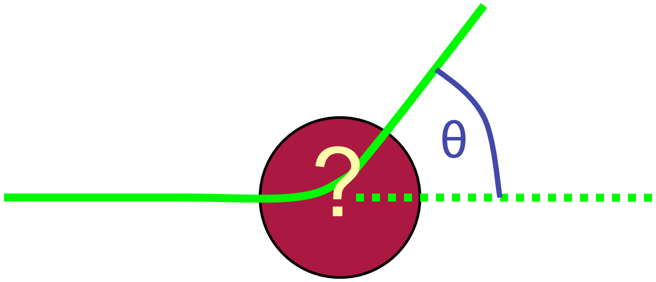
http://www.hep.ph.ic.ac.uk/~tapper/lecture.html

Outline

- We'll start with some history of the structure of matter and scattering experiments.
- End up at the parton model and key results that are proof for quarks and gluons.
- Add interactions (QCD) to the parton model and see how this looks.
- Recent developments (HERA) and state of the art in structure of the proton.
- Application to some LHC processes.

Rutherford scattering

Rutherford taught us the most important lesson: use a scattering process to learn about the structure of matter



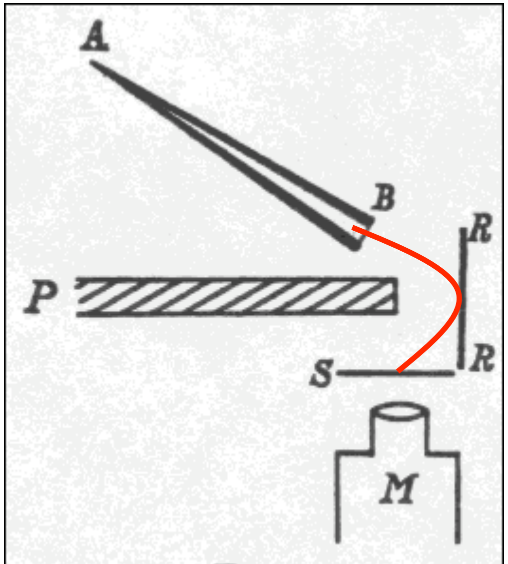


Fig. 1.

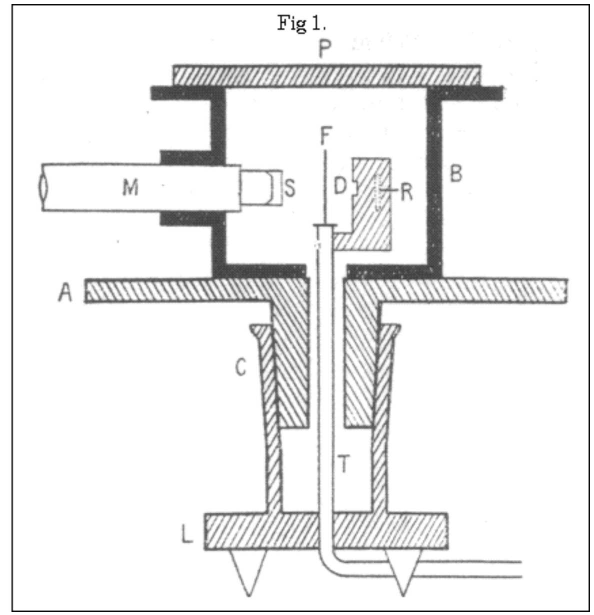
H. Geiger and E. Marsden observed that α-particles were sometimes scattered through very large angles.

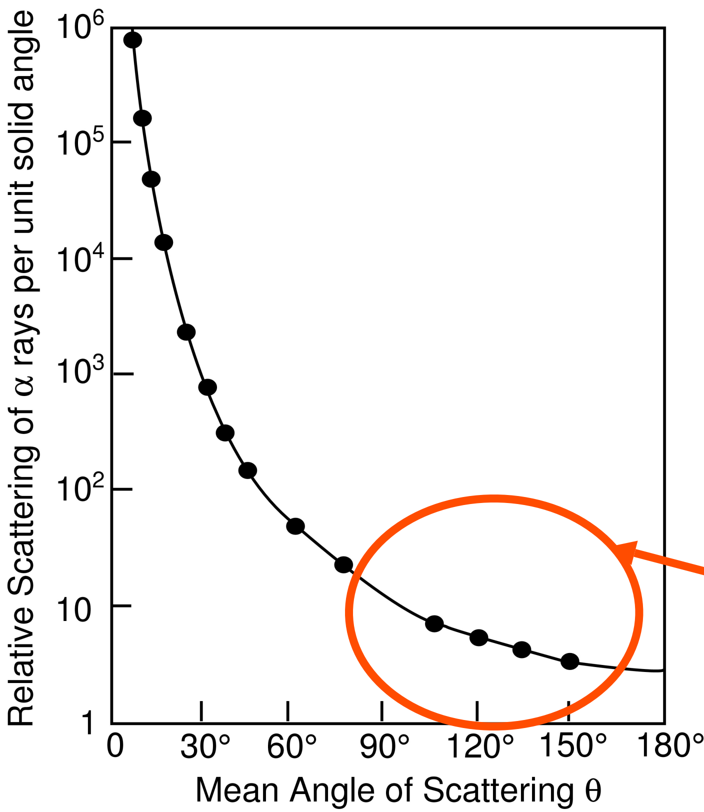
Rutherford interpreted these results as due to the coulomb scattering of the α-particles with the atomic nucleus:

$$\sigma(\theta) = \frac{z^2 Z^2 e^4}{16 E^2} \frac{1}{\sin^4 \frac{1}{2}\theta}$$

Rutherford scattering

In a subsequent experiment Geiger and Marsden verified Rutherford's prediction





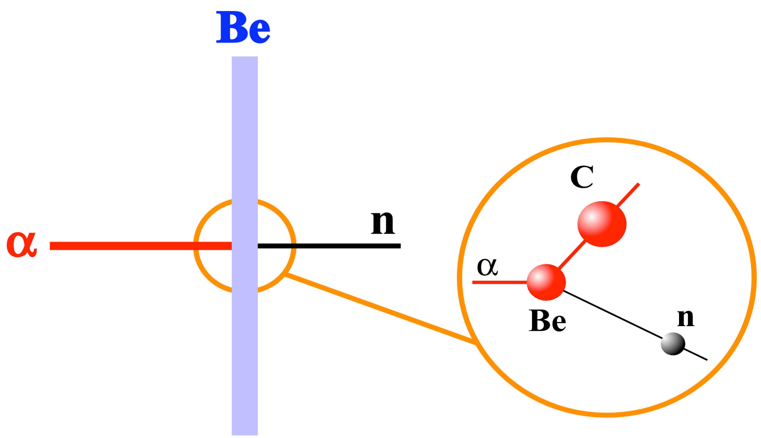
Discovery of atomic nucleus

Discovery of the neutron

Composition of the nucleus remained a mystery until….

Discovery of neutron

(Chadwick 1932)



By this time many things known about the nucleus, for example:

$R = r_0 x A^{1/3}$ fm with $r_0 = 1.45$ fm

$\rho_m = 0.08$ nucl/fm^3 and $\rho_c = (Z/A) x 0.08$ (prot. charges)/fm^3

Nucleus form factor

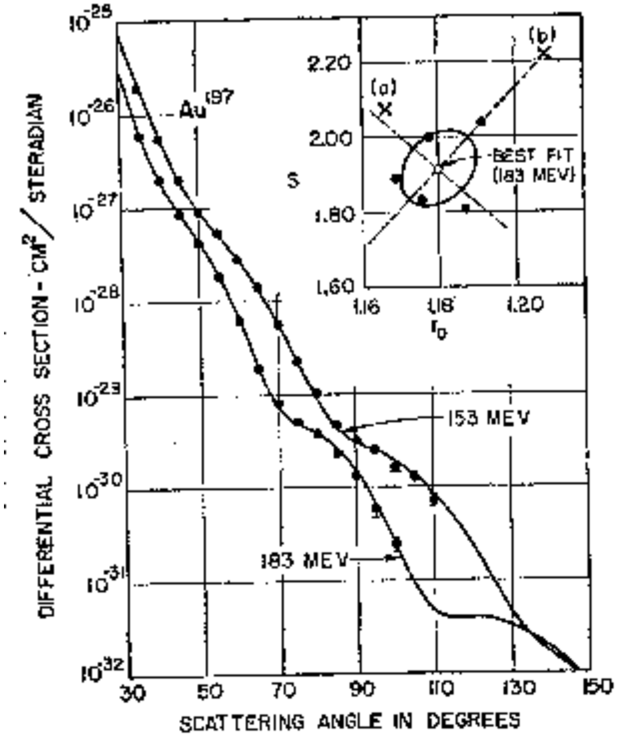
Calculations for the structure of the nucleus. Depending on the assumed charge distribution.

$$\sigma_M(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}$$

Mott scattering

$$\sigma_s(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta} \left| \int_{\substack{\text{nuclear}\\ \text{volume}}} \rho(r) e^{i\mathbf{q}\cdot\mathbf{r}} d\tau \right|^2$$

$$\sigma_s(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta} \left[\int_0^\infty \rho(r) \frac{\sin qr}{qr} 4\pi r^2 dr \right]^2.$$



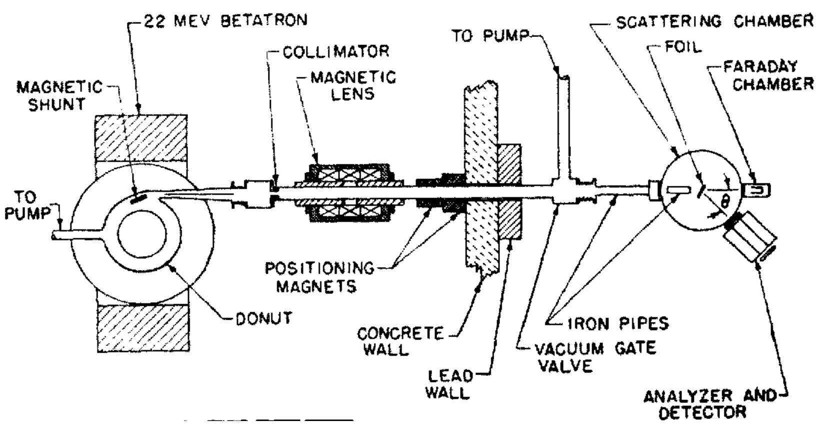
$$F = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin(qr) r dr$$

Nucleus form factor

First evidence for elastic electron-nucleus scattering

Scattering of 15.7-Mev Electrons by Nuclei*

E. M. Lyman, A. O. Hanson, and M. B. Scott†
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 3, 1951)



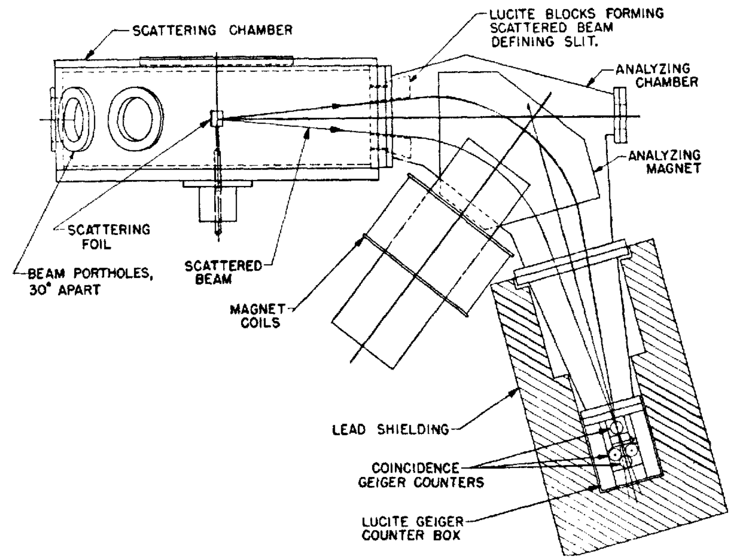
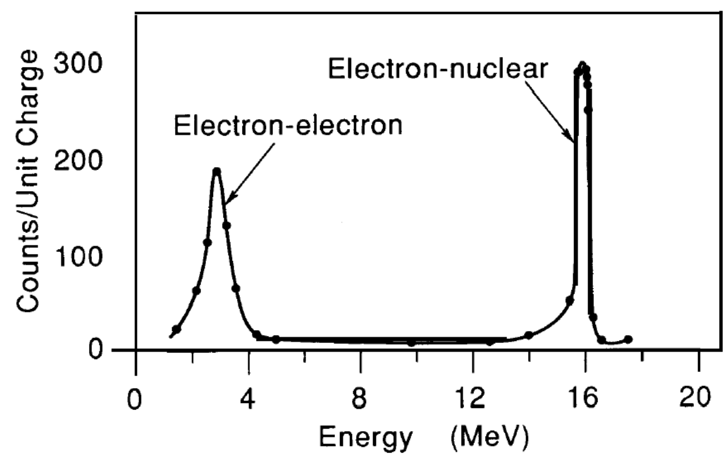


Fig. 2. Scattering chamber.



Point-like electron probe better than Rutherford's α-particles

Probing power: $\lambda \propto 1/E$

Linacs and Stanford

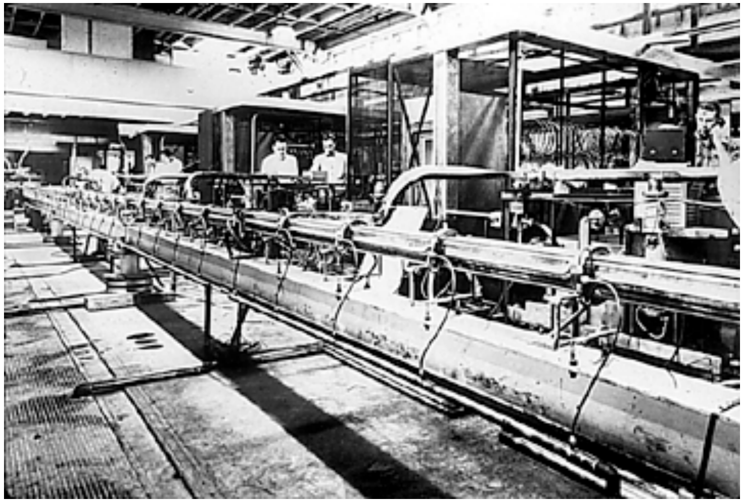
Russell and Sigurd Varian

In the mid '30s the Varian brothers (working as research assistants at the microwave department of Stanford University) developed the klystron exploiting a particular electromagnetic cavity (Rhumbatron) developed by W. Hansen

This device played a fundamental role in the leading role that the Stanford HEP was going to play in Linac development…

Under the direction of E. Ginzton the Stanford started the construction of "small" scale linacs (MARKI,II,III).

The half-completed MARK III was instrumental to R. Hofstadter's e-N e-p elastic scattering experiments…

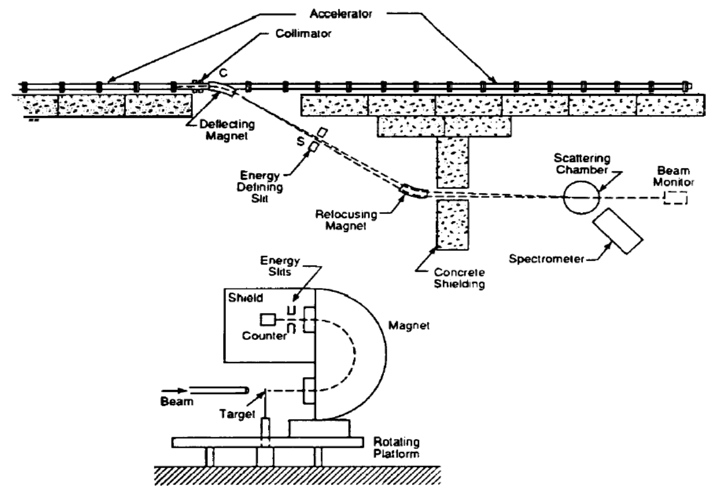


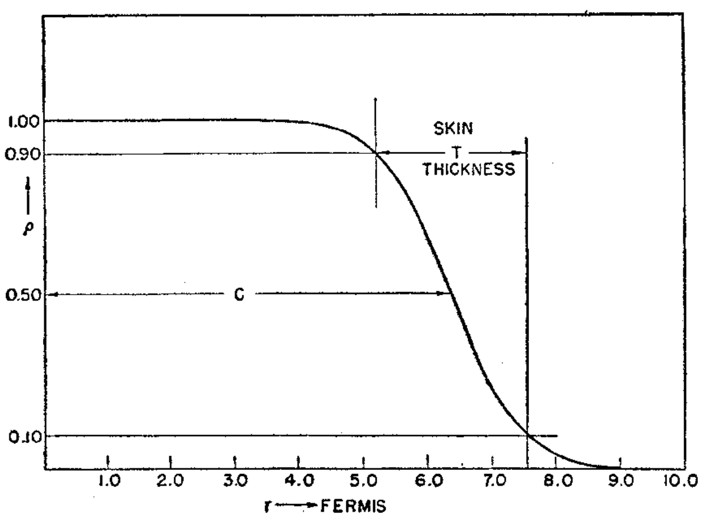
R. Hofstadter: e-N elastic scattering

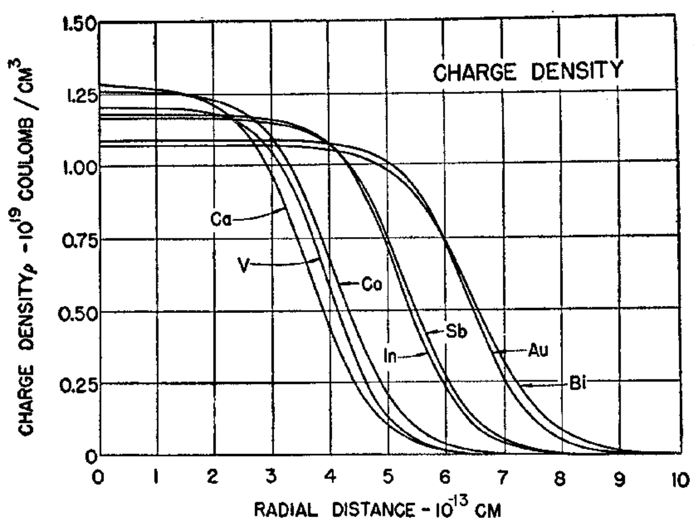
Fermi model

$$\rho(r) = \frac{\rho_1}{\exp[(r-c)/z_1]+1}.$$

C= (1.07±0.02)xA$^{1/3}$ fm
t = (2.4±0.3) fm



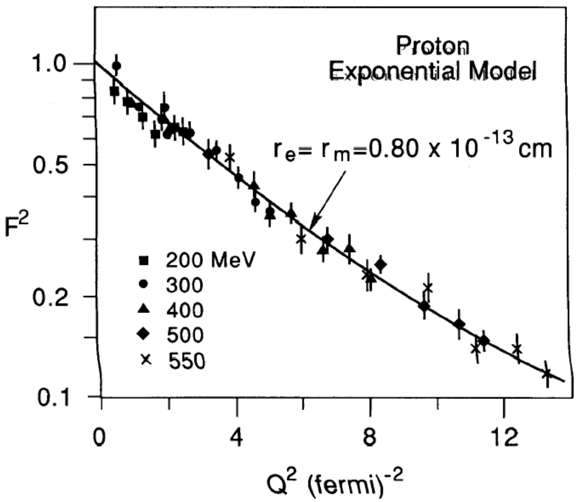




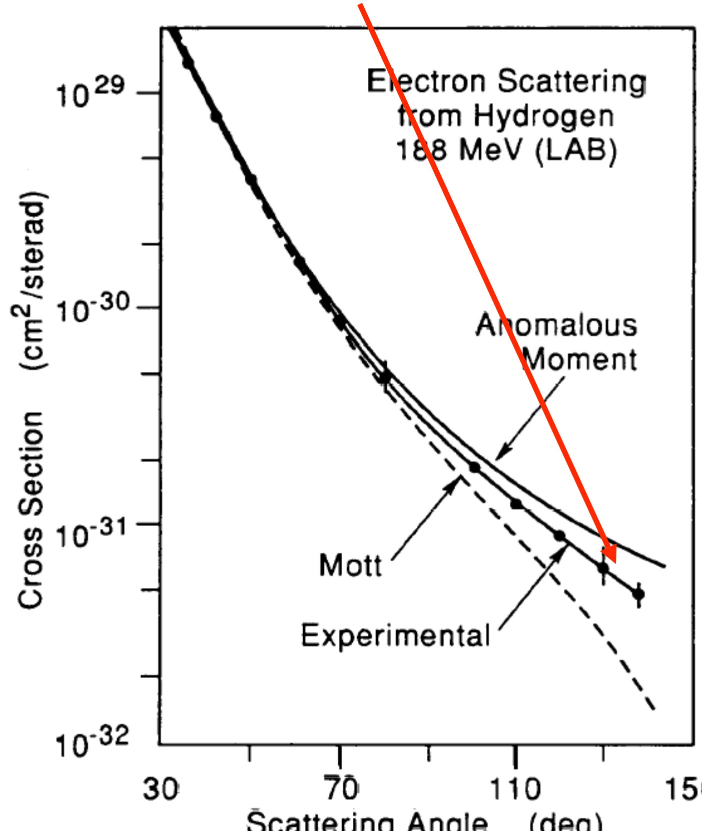
R. Hofstadter: e-p and e-d elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_0^2 \sin^4 \theta/2} \cdot \cos^2 \theta/2 \cdot \frac{E'}{E_0} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \theta/2 \right]$$

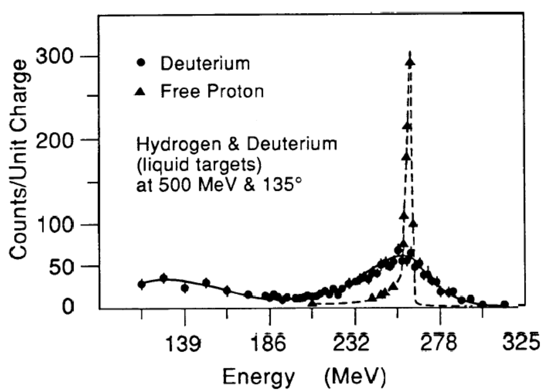
First determination of the proton form factor



Protons are composite particles!



Deuterium smeared out by Fermi energy



R. Hofstadter: 1961 Nobel lecture

"As we have seen, the proton and neutron, which were once thought to be elementary particles are now seen to be highly complex bodies. It is almost certain that physicists will subsequentely investigate the constituent parts of the proton and neutron - the mesons of one sort or another. What will happen from that point on ? One can only guess at future problems and future progress, but my personal convinction is that the search for ever-smaller and ever-more-fundamental particles will go on as Man retains the curiosity he has always demonstrated"

So what happened next?

SLAC and the M-project

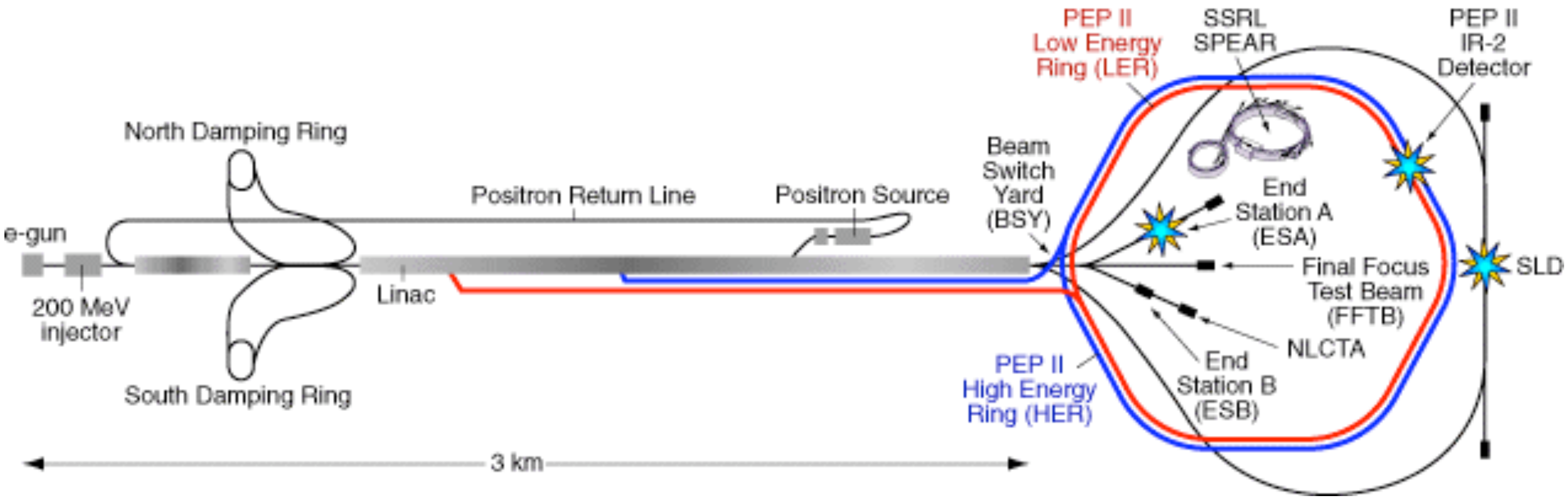
On april 10, 1956, Stanford staff met in Prof. W. Panofsky's home to discuss Hofstadter's suggestion to build a linear accelerator that was at least 10 times as powerful as the Mark III. This idea was called "The M(onster)-project" because the accelerator would need to be 2 miles long!!

- 1957 A detailed proposal was presented
- 1959 Eisenhower said yes
- 1961 Congress approved the project ($114M)

One year later construction started

While excavating SLAC the workers discovered a nearly complete skeleton of a 10-foot mammal, Paleoparadoxia, which roamed earth 14 millions years ago…

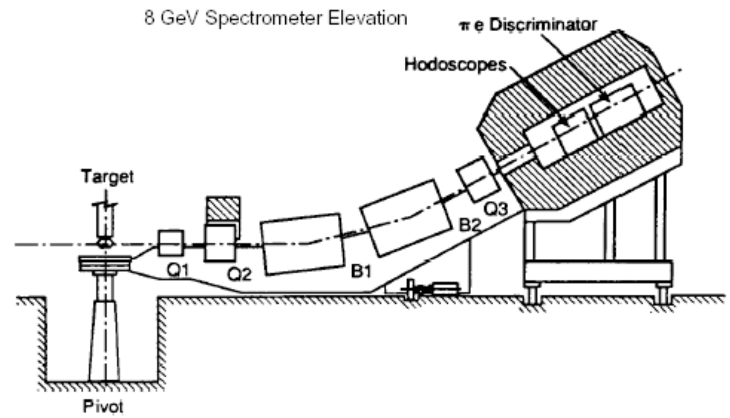
SLAC today

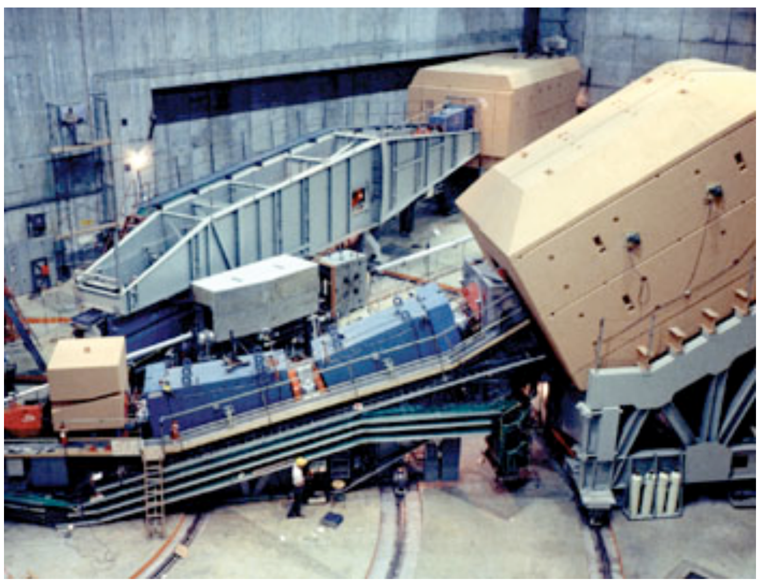


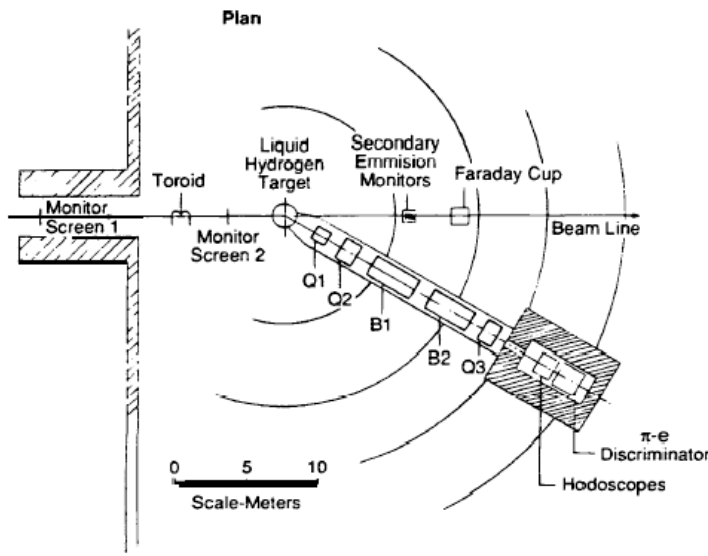
Stanford linear collider still operates today for BaBar

The SLAC-MIT experiment

Taylor, Friedman and Kendall

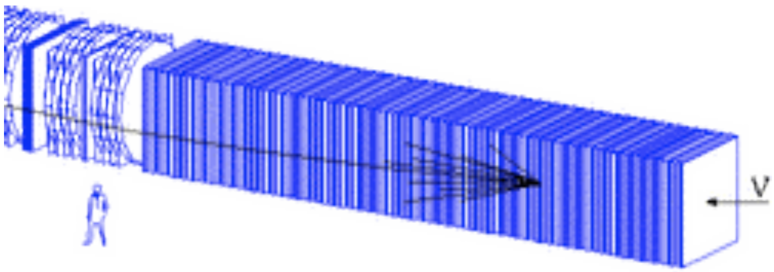


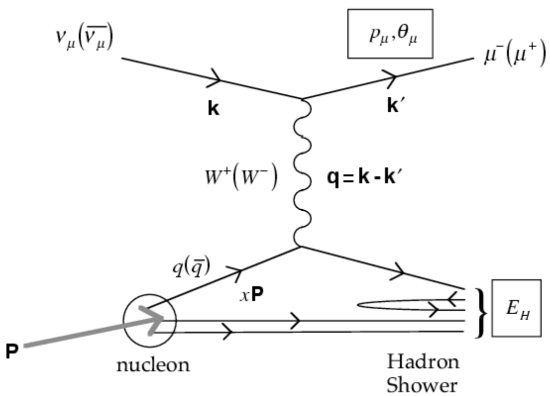


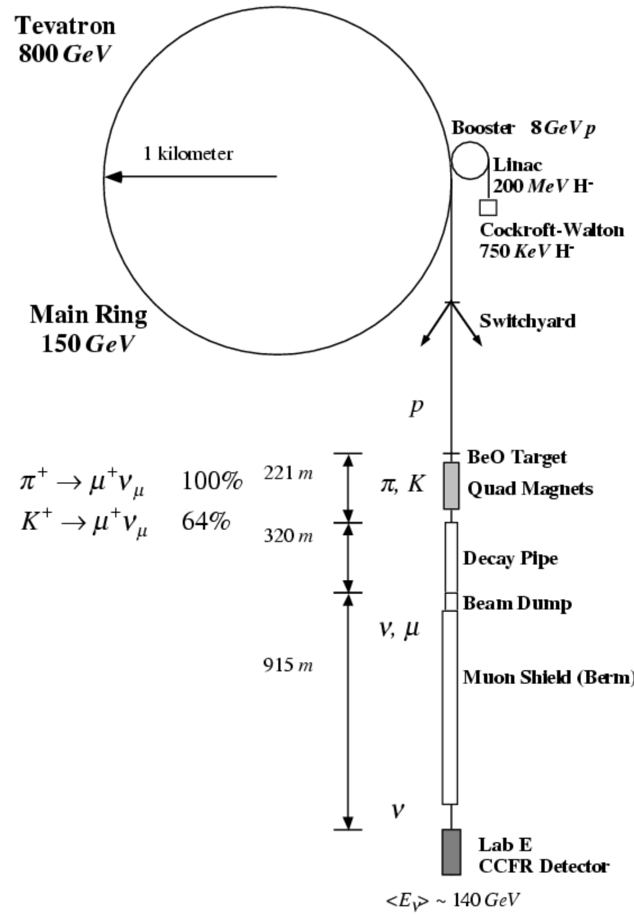


Neutrino scattering

For example the CCFR experiment at Fermilab







$$\frac{d^2\sigma^{\nu(\overline{\nu})}}{dx\,dy} = \frac{G_F^2 ME}{\pi}\left[\left(1-y-\frac{Mxy}{2E}\right)F_2^{\nu(\overline{\nu})} + \frac{y^2}{2}2xF_1^{\nu(\overline{\nu})} \pm y\left(1-\frac{y^2}{2}\right)\nu F_3^{\nu(\overline{\nu})}\right]$$

What do we know so far?

- Now we know from Rutherford about the atomic nucleus
- From Chadwick about neutrons
- From Hofstadter that protons and neutrons are composite particles
- High energy electron-nucleon and neutrino-nucleon experiments to study proton and neutron structure
  - Results will be very important

- But first we need to know a little theory…. otherwise we won’t know what to look for.

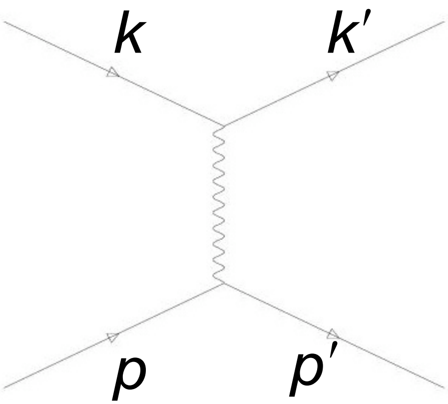
Theory - the state of play

- Zoo of hadrons discovered in bubble chamber experiments etc.
- Gell-Mann and Zweig proposed quarks and aces to classify the animals
  - Simple static model (for example p=uud)
  - Fractional charge
- Parton model by Feynman
  - Non-interacting point-like particles
  - Valence and sea structures

Reminder of electron-muon scattering

Start by considering scattering QED process of scattering two distinct spin-1/2 charged particles

The matrix element is given by:



$$M = \frac{g_{EM}^2}{(k-k')^2}\left[\bar{u}(k')\gamma_\mu u(k)\right]_{electron}\left[\bar{u}(p')\gamma^\mu u(p)\right]_{muon}$$

same form with either the electron or muon mass appearing where appropriate. Squaring this to find the cross section gives the result:

$$|M|^2 = \frac{\alpha^2}{q^4} L_{\mu\nu electron} L^{\mu\nu}_{muon}$$

Electron-muon scattering

$L_{\mu\nu}$ has the form: $$L^{\mu\nu} = \left[\bar{u}^{*}\gamma^{\mu}u^{*}\right]\left[\bar{u}\gamma^{\nu}u\right]$$

This can be evaluated using standard trace techniques. The 4-momentum transfer between electron and muon is written as:

$$q^{\mu} = (k - k')^{\mu} = (p' - p)^{\mu}$$

The final result for the electron $L_{\mu\nu}$ is

$$L^{\mu\nu}_{electron} = 4k^{\mu}k^{\nu} - 2q^{\mu}k^{\nu} - 2k^{\mu}q^{\nu} + g^{\mu\nu}q^{2}$$

with a similar result for the muon with k replaced by p and q by -q

Electron-muon scattering

To find the cross section, the electron and muon tensors must be contracted together. This is made easier by the observation that:

$$q_\mu L^{\mu\nu} = q_\nu L^{\mu\nu} = 0$$

This means that the terms in q_μ, q_ν in one of the tensors can be ignored. This property is a requirement from current conservation (see Aitchison & Hey, chap. 3).

Putting everything together, we get:

$$|M|^2 = \frac{\alpha^2}{q^4}\left\{4(2p\cdot k)^2 + 4(2p\cdot k)q^2 + 2q^4 + 4q^2(m^2+M^2)\right\}$$

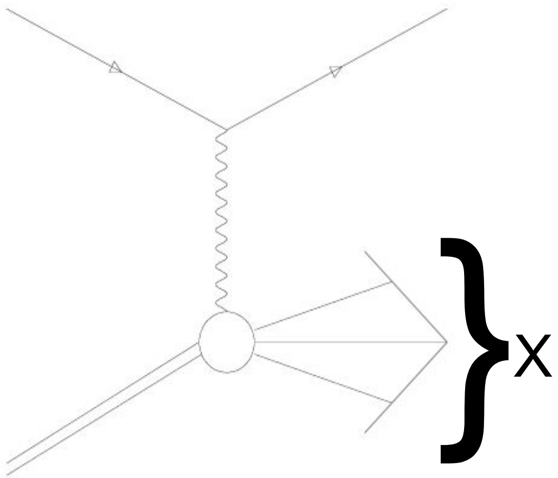
where m and M are the electron and muon masses

Inelastic lepton-nucleon scattering

We are interested in the inelastic process lN → l′X

l and l′ are leptons, N is the nucleon

X is some hadron state whose mass is generally denoted by W



Look first at the kinematics of this process

Denote lepton momenta as k and k′ as before and the 4-momentum transferred by the virtual photon to the hadronic system by q = k - k′

The initial nucleon 4-momentum is p and the total for the final state hadrons is p′ = p + q

Kinematics of inelastic lN scattering

The final state is described by:

- The 3-momentum of the lepton plus its mass
- The 4-momentum of the hadronic system

Constraints from energy-momentum conservation mean that these 7 independent quantities can be reduced to 3, one of which can be taken as the overall azimuthal angle

- Hence we can see that the final state is described by 2 independent kinematic quantities, which are taken to be the Lorentz invariants:

$$q^2 = q^\mu q_\mu, \quad p \cdot q = p^\mu q_\mu$$

Momentum transferred²

Related to scattering angle

Inelastic lepton-nucleon scattering

- Start by considering EM interactions where l = electron/muon
- We can't calculate anything for the diagram on the earlier slide because we don't know how to treat the hadronic part
- However, we can find an expression that takes into account what we do know about the leptonic part
  - Based on the cross section for electron-muon scattering, we might expect something similar in this case
  - The whole point of DIS experiments is that leptons & EM interactions which are well understood are used to probe the hadronic part (which isn't).

Inelastic lepton-nucleon scattering

The result for lepton-hadron scattering is:

$$\left| M \right|^2_{DIS} = \frac{\alpha^2}{q^4} L_{\mu\nu,electron} W^{\mu\nu}_{hadron} \cdot 4\pi M_N$$

$W^{\mu\nu}$ is an unknown tensor function of the kinematic variables which is to be determined by experiment

The factor $4\pi M_N$ is included to give what Halzen & Martin call the “standard convention” for the normalisation of $W^{\mu\nu}$

$W^{\mu\nu}$ represents the unknown physics at the photon-hadron vertex. It is therefore a function of the photon 4-momentum q^{μ} and the initial nucleon 4-momentum p^{μ}

The hadronic tensor

The most general form of the hadronic tensor is:

$$W^{\mu\nu} = W_1(q^2, p\cdot q)\left\{-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right\} + \frac{W_2(q^2, p\cdot q)}{M_N^2}\left\{p^\mu - \frac{p\cdot q}{q^2}q^\mu\right\}\left\{p^\nu - \frac{p\cdot q}{q^2}q^\nu\right\}$$

The form of the leptonic tensor is again restricted by current conservation. The squared matrix element for DIS is:

$$\left|\mathrm{M}\right|^2_{DIS} = 4\pi M_N \frac{\alpha^2}{q^4}\left\{W_1(-2q^2 - 4m^2) + \frac{W_2}{M_N^2}\left((2p\cdot k)^2 - (2p\cdot q)(2p\cdot k) + q^2 M_N^2\right)\right\}$$

This decomposition of $W^{\mu\nu}$ introduces the conventional definition of the arbitrary functions $W_{1,2}$. They have the same dimensions (energy^{-1}). In the proton rest frame the angular dependence of the cross section separates into a W_1 and a W_2 term (see Halzen & Martin, chap. 8 for details)

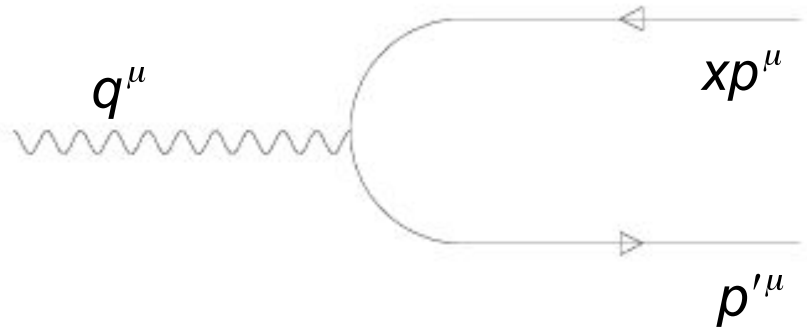
The parton model

Look at inelastic scattering from a different point of view:

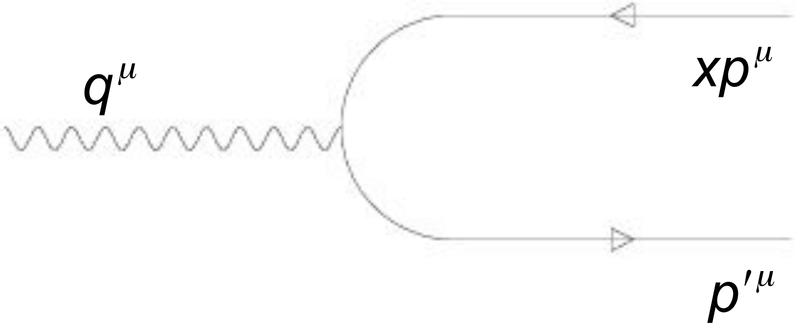
- Let us suppose that the underlying process is elastic scattering of the electron off some constituent of the nucleon
  - Call this constituent a "parton" & assume it carries a momentum fraction x of the nucleon's 4-momentum

  (slightly suspect as it implies parton mass = xM_N. However assume momenta are large enough that M_N can be neglected)

Look at virtual photon-parton interaction in the Breit or "brick wall" frame:



The parton model



In the Breit frame:

- Photon energy is zero
- Photon & parton momenta directed along z-axis

or:

$$q^{\mu} = (0,0,0,\underline{q})$$

$$xp^{\mu} = (xE,0,0,x\underline{p})$$

Final state parton will have same energy (xE), so the magnitude of the momentum remains the same:

$$p'^{\mu} = (xE,0,0,-x\underline{p})$$

The parton model

Imposing momentum conservation means that:

$$\underline{q} = -2x\,\underline{p}$$

It then follows that

$$q^2 = -4x^2\,\underline{p}^2$$

$$p \cdot q = 2x\,\underline{p}^2$$

$$x = \frac{-q^2}{2p \cdot q}$$

We have expressed x in terms of our Lorentz invariant quantities, q^2 and p·q. This is a frame-independent expression, although we have used a specific frame to derive it

The parton model

- The cross section for electron-parton scattering will depend on the parton spin.
- If partons are Dirac particles, it will be the same as for electron-muon scattering, multiplied by the squared charge of the parton
- Assume there are different types of parton, with parton q have charge e_q and its momentum distribution in the proton given by $f_q(x)$
  - $W^{\mu\nu}$ can then be found by summing the contribution from all different types of partons
  - Strictly speaking we also need to integrate over the outgoing parton (hadron system) phase space (takes into account the fact that we are making inclusive measurements)
  - This introduces a Dirac delta-function to ensure the relation for x

The parton model

Including the correct normalisation for $W^{\mu\nu}$, the parton model prediction is:

$$4\pi M_N \cdot W^{\mu\nu} = \sum_q f_q(x) L^{\mu\nu}_{parton} 2\pi\delta(Q^2 - 2xp \cdot q)$$

$$= 2\pi \sum_q f_q(x) L^{\mu\nu}_{parton} \delta(1 - \frac{2xp \cdot q}{Q^2})$$

Or

$$|M|^2_{DIS} = \frac{2\pi}{Q^4} \sum_q f_q(x) |M|^2_{parton}$$

$|M|^2_{parton}$ is the squared QED matrix element for lepton-parton scattering

The parton model

Starting from the formula for the matrix element squared for electron-muon scattering, it is possible (try it for yourself !) to show that:

$$|M|^2_{parton} = e^2_{parton}\alpha^2 \left\{ 2 + 4\left(\frac{1-y}{y^2}\right) - \frac{4(m^2 + x^2 M_N^2)}{Q^2} \right\}$$

where $Q^2 = -q^2$ and y is a dimensionless Lorentz invariant quantity defined by

$$y = \frac{p \cdot q}{p \cdot k} = \frac{Q^2}{2xp \cdot k}$$

This means that the matrix element squared for DIS can be written as:

$$|M|^2_{DIS} = 4\pi M_N \alpha^2 \left\{ \frac{2W_1}{Q^2} + \frac{W_2}{x^2 M_N^2}\left(\frac{1-y}{y^2}\right) - \frac{4W_1 m^2 + W_2 Q^2}{Q^4} \right\}$$

The parton model

By comparing the two equations for the DIS squared matrix element, it can be shown that (again, try it for yourself !):

$$2M_N W_1 = \sum_q e_q^2 f_q(x)$$

$$\frac{p \cdot q}{M_N} W_2 = \sum_q e_q^2 x f_q(x)$$

It is usual to replace these functions by:

$$F_{2_{parton}}(x) = \frac{p \cdot q}{M_N} W_2 = 2xM_N W_1 = \sum_q e_q^2 x f_q(x)$$

Note: The parton model predicts that measurements of W_1 & W_2 as a function of q^2 and p•q will give results which depend only on their ratio. Note also that W_1 & W_2 are directly related (the Callan-Gross relation) - this is a direct result of partons having spin-1/2. Other values for the spin would give different relationships

Parton model cross section for DIS

The parton model result for the deep inelastic ep scattering cross section is:

$$\frac{\partial^2\sigma^{ep}}{\partial x\partial y} = \frac{4\pi\alpha^2}{Q^4}(s - M_N^2)\left\{\frac{1}{2}(1 + (1-y)^2) - \frac{x^2y^2M_N^2}{Q^2}\right\}F_2^{ep}(x)$$

The term in the electron mass has been dropped and any effects from weak interactions have been ignored

The F_2 measured in deep inelastic electron scattering gives the sum of the quark distribution functions weighted by the square of the quark charge:

$$F_2^{ep} = \frac{4}{9}(xf_u^p + xf_{\bar{u}}^p + xf_c^p + xf_{\bar{c}}^p) + \frac{1}{9}(xf_d^p + xf_{\bar{d}}^p + xf_s^p + xf_{\bar{s}}^p)$$

The heavier quarks have been neglected

Parton model cross section for DIS

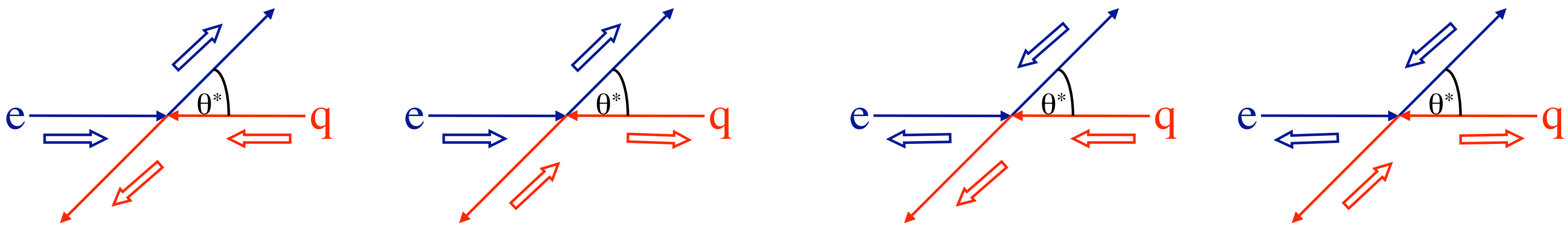
At high energies, the second term in the double differential cross section disappears. In this approximation the result is the average of two contributions for scattering particles of the same and opposite helicities. The variable y is related to the electron scattering angle θ^* in the electron-parton CM system by:

$$y = \sin^2 \frac{\theta^*}{2}$$

This means that y=0 corresponds to forward scattering and y=1 to backward scattering. The term for scattering particles of opposite helicity is proportional to $(1 - y)^2$ and vanishes in the backward direction by angular momentum conservation.

Parton model cross section for DIS

Consider the lepton-quark centre-of-mass frame:



M∝1 | M∝(1+cosθ*) | M∝(1+cosθ*) | M∝1

$$M(e_L+q_L)^2 = M(e_R+q_R)^2 \propto 1$$

$$M(e_L+q_R)^2 = M(e_R+q_L)^2 \propto (1+\cos\theta^*)^2 \propto \left(1-\sin^2\tfrac{\theta^*}{2}\right)^2 \propto (1-y)^2$$

Helicity conserved (m_e and m_q << E) so angular momentum conservation gives the suppression factor $(1\text{-}y)^2$ for scattering of particles with opposite helicities.

Parton model charged current DIS

Similar expressions can be written for other processes. For example, consider charged current neutrino-proton scattering $\nu_\mu p \to \mu^- X$, only d-type quarks and u-type antiquarks contribute.

The parity-violating nature of the weak interaction means that the angular distributions predicted for interactions involving quarks and antiquarks are different. The cross section is:

$$\frac{\partial^2 \sigma^{\nu p}}{\partial x \partial y} = \frac{G_F^2}{\pi}(s - M_N^2)\left\{(xf_d^p + xf_s^p) + (xf_{\bar{u}}^p + xf_{\bar{c}}^p)(1-y)^2\right\}$$

This can be written in the form:

$$\frac{\partial^2 \sigma^{\nu p}}{\partial x \partial y} = \frac{G_F^2}{\pi}(s - M_N^2)\left\{\frac{1}{2}\left(1 + (1-y)^2\right)F_2^{\nu p}(x) + \frac{1}{2}\left(1 - (1-y)^2\right)xF_3^{\nu p}(x)\right\}$$

The component proportional to F_2 ➔same angular distribution as for eq and xF_3 term contains additional parity-violating part

Parton model charged current DIS

In terms of quark momentum distributions, F_2 and xF_3 are:

$$F_2^{\nu p} = \left((xf_d^p + xf_s^p) + (xf_{\bar{u}}^p + xf_{\bar{c}}^p)\right)$$

$$xF_3^{\nu p} = \left((xf_d^p + xf_s^p) - (xf_{\bar{u}}^p + xf_{\bar{c}}^p)\right)$$

The momentum distributions for the remaining quark flavours contribute to the analogous expressions for antineutrino-proton scattering.

The neutrino-neutron cross section is the same as the antineutrino-proton one, assuming isospin invariance

If we have a target consisting of equal numbers of p & n, F_2 gives an unweighted sum of the distributions of all quark flavours (in the proton):

$$F_2^{\nu N} = \left((xf_d^p + xf_s^p) + (xf_d^n + xf_s^n) + (xf_{\bar{u}}^p + xf_{\bar{c}}^p) + (xf_{\bar{u}}^n + xf_{\bar{c}}^n)\right)$$

$$= \left((xf_d^p + xf_s^p) + (xf_u^p + xf_c^p) + (xf_{\bar{u}}^p + xf_{\bar{c}}^p) + (xf_{\bar{d}}^p + xf_{\bar{s}}^p)\right)$$

$$= \sum_q x(f_q + f_{\bar{q}})$$

Parton model charged current DIS

By assuming isospin invariance:

$$xF_3^{\nu N} = \left((xf_d^p + xf_s^p) + (xf_u^p + xf_c^p) - (xf_{\bar{u}}^p + xf_{\bar{c}}^p) - (xf_{\bar{d}}^p + xf_{\bar{s}}^p)\right)$$

$$= \sum_q x(f_q - f_{\bar{q}})$$

In summary, measurements made with different beams & targets are sensitive to different combinations of the quark distribution functions.

Important results come from integrating the measured structure functions over x, eg. integration of F_2 gives total momentum fraction carried by quarks, while integration of F_3 should give the number of valence quarks

Summary

We covered:

- Some fun history
- The key experiments
- The predictions of the parton model

The parton model assumes:

- Non-interacting point-like particles
  $\rightarrow$ Bjorken scaling i.e. $F_2(x,Q^2)=F_2(x)$
- Fractional charges (if partons=quarks)
- Spin 1/2
- Valence and sea quark structure (sum rules)

Makes key predictions that can be tested by experiment

We'll see the experimental measurements next time…