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QCD and deep inelastic scattering

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Slides available at:

http://www.hep.ph.ic.ac.uk/~tapper/lecture.html

Reminder

In the first lecture we covered the predictions of the parton model

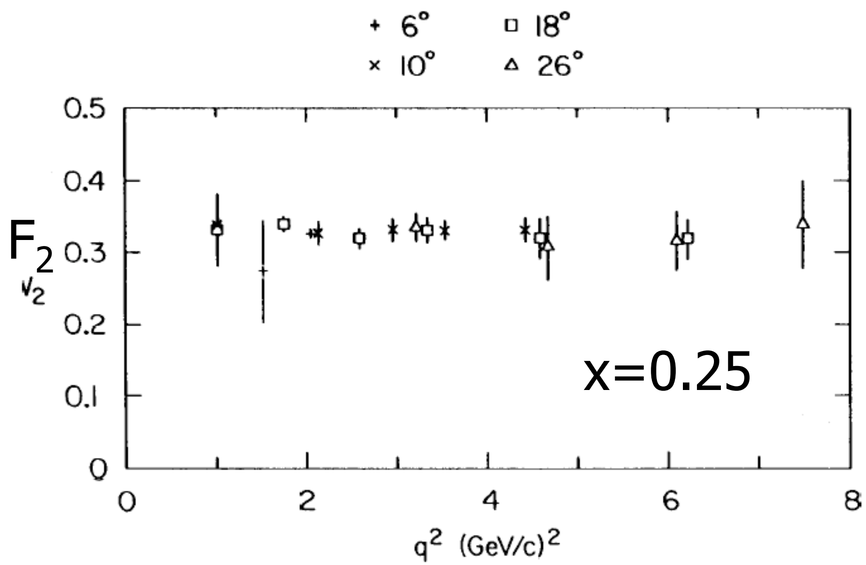
The parton model assumes

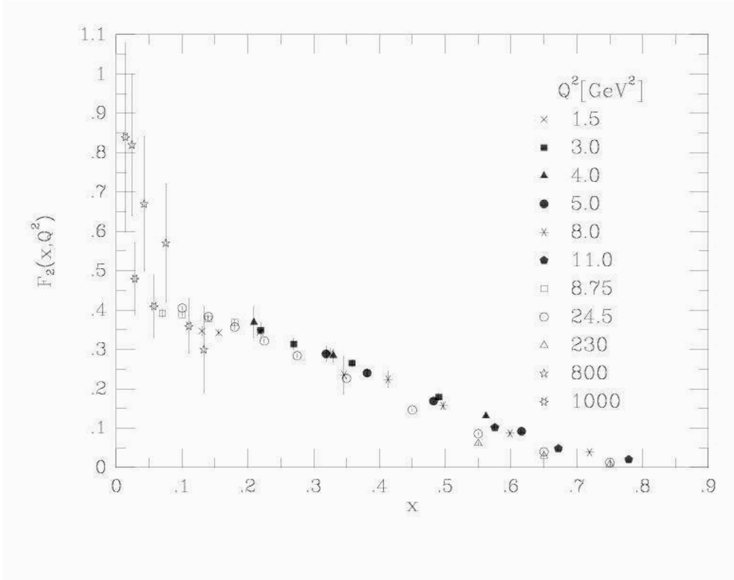
- Non-interacting point-like particles
  - → Bjorken scaling i.e. $F_2(x,Q^2)=F_2(x)$
- Fractional charges (if partons=quarks)
- Spin 1/2
- Valence and sea quark structure (sum rules)

Makes key predictions that can be tested by experiment

Deep inelastic scattering results

Bjorken scaling $F_2(x,Q^2)=F_2(x)$



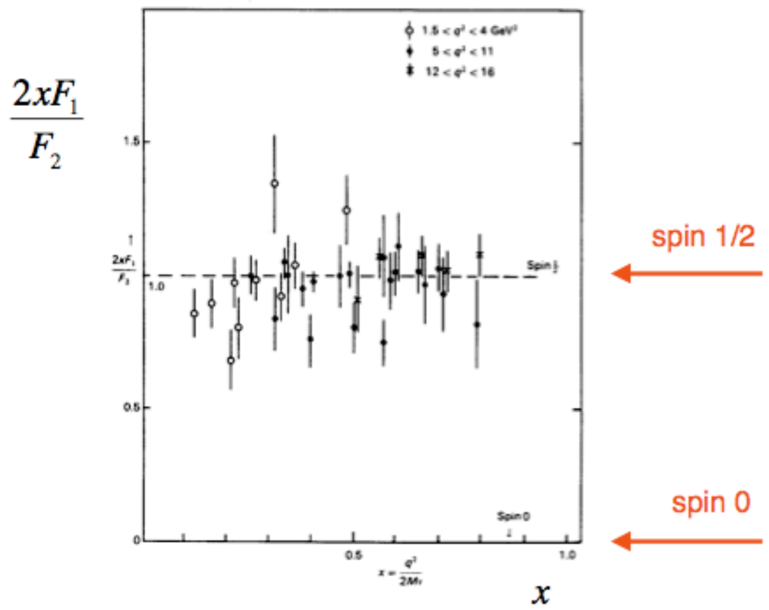


→ Scaling is observed so quarks point-like

Deep inelastic scattering results

F_1 and F_2 relationship derived from the Dirac equation

The Callan-Gross relation $F_2 = 2xF_1$



→ Quarks are spin 1/2 fermions

Key predictions

A good check of the fractional charges assigned to the quarks is given by the following.

Since

$$F_2^{\nu N} = \sum_q x\left[q(x) + \overline{q}(x)\right]$$

$$F_2^{ep} = \sum_q x \cdot e_q^2\left[q(x) + \overline{q}(x)\right]$$

and

$$F_2^{ep} = x\left[\tfrac{4}{9}(u + \overline{u}) + \tfrac{1}{9}(d + \overline{d})\right] \qquad F_2^{en} = x\left[\tfrac{4}{9}(d + \overline{d}) + \tfrac{1}{9}(u + \overline{u})\right]$$

Adding F_2^{ep} and F_2^{en} to F_2^{eN} gives the relation

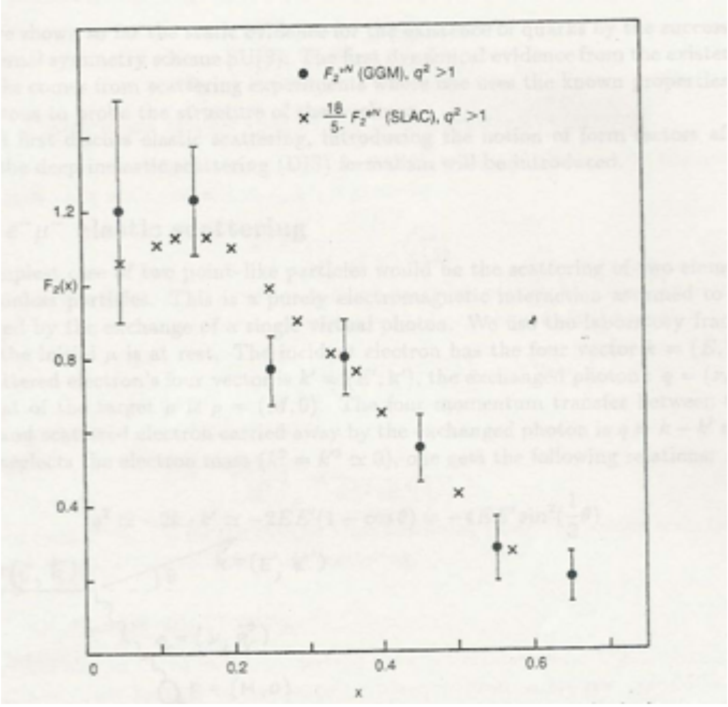
$$\boxed{F_2^{eN}(x) = \tfrac{5}{18} F_2^{\nu N}(x)}$$

Approximate since only consider up and down quarks

Deep inelastic scattering results

Comparison of eN and νN F_2

$$F_2^{eN}(x) = \frac{5}{18} F_2^{\nu N}(x)$$



Key predictions

A check of the valence structure assigned to the quarks is given by the following.

Since

$$F_2^{ep} = x\left[\tfrac{4}{9}(u+\overline{u}) + \tfrac{1}{9}(d+\overline{d})\right] \qquad F_2^{en} = x\left[\tfrac{4}{9}(d+\overline{d}) + \tfrac{1}{9}(u+\overline{u})\right]$$

Subtracting F_2^{ep} and F_2^{en} gives the relation

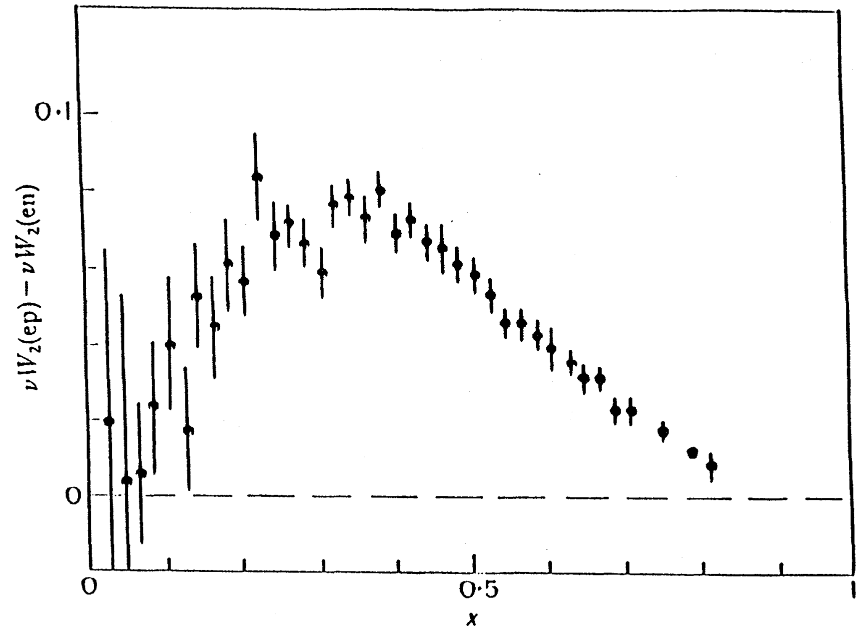
$$F_2^{ep} - F_2^{en} = \frac{1}{3} x(u_v(x) - d_v(x))$$

So if nucleons consist of three bound valence quarks, we would expect this measurement to come out at 1/3

Deep inelastic scattering results

Comparison of ep and en F_2

$$F_2^{ep} - F_2^{en} = \frac{1}{3} x(u_v(x) - d_v(x))$$



Key predictions

From neutrino scattering:

$$F_2^{\nu N} = \sum_q x\left[q(x) + \overline{q}(x)\right] \qquad xF_3^{\nu N} = \sum_q x\left[q(x) - \overline{q}(x)\right]$$

So summing F_2 over all partons gives the total fraction of the protons momentum carried by the partons

$$\int_0^1 F_2^{\nu N}(x)dx = \int_0^1 x\left(q(x) + \overline{q}(x)\right)dx = 0.44$$

So about half the momentum of the proton is not carried by the quarks! (at $Q^2 \sim 10$ GeV)

→ Where did the rest of the momentum go?

Summary of the naïve parton model

From DIS using electron/muon/neutrino beams

- Structure functions show Bjorken scaling behaviour
- $F_2=2xF_1$ as expected for spin 1/2 quarks
- Comparison of neutrino and electron scattering confirms quark charge assignments
- Comparison of proton and neutron data confirms valence structure
- Only ~50% of proton momentum carried by quarks, rest by something electrically neutral and not probed by photon?
- Also other qualitative tests passed (sum rules etc.)

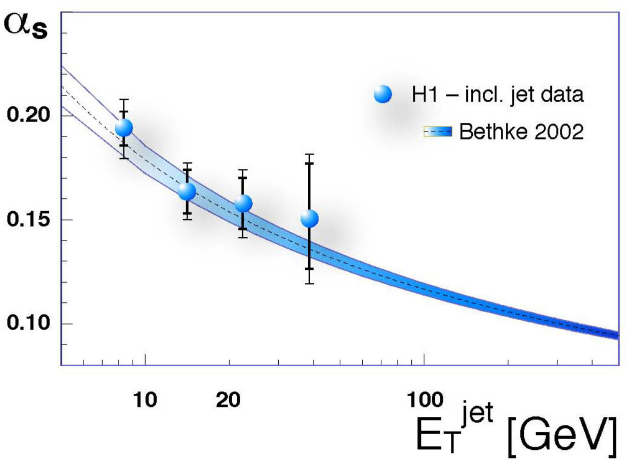
- All remarkably successful and convincing proof of quarks and gluons
- Actually most of these are only approximate
- Now time to add interactions to the model →

QCD

- Gauge field theory of the interactions of coloured quarks
  - Based on QED
  - Instead of the photon → 8 coloured gluons

- Leads to many phenomenological differences

- Probably the most important:
  - In QED coupling constant (α) increases with decreasing distance (or increasing Q^2)
  - In QCD coupling constant (α_S) decreases with decreasing distance (or increasing Q^2)



QCD picture of proton structure

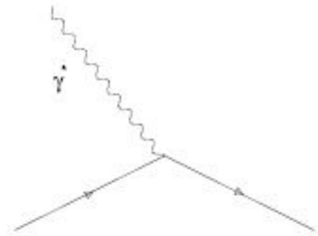
- QCD picture of proton structure:
  - Valence quarks are bound together by gluon exchange
  - The virtual gluons can fluctuate into quark-antiquark pairs, producing the "sea"
  - Sea quarks are found predominantly at low x
  - Scaling is violated because as the proton is probed with increasing resolution (or Q^2), the results of QCD processes such as gluon emissions, quark-antiquark pair production become "visible"

- QCD allows us to calculate how the structure functions change with Q^2

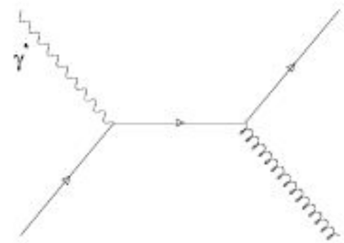
QCD corrections to the parton model

In the parton analysis, the simple two-body QED process eq $\rightarrow$ eq was used to analyse the DIS reaction.

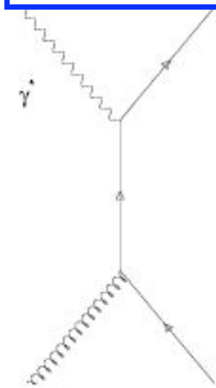
The matrix element was factorised into a leptonic part (e $\rightarrow$ eγ^*) and a hadronic part (γ^*q $\rightarrow$ q). This now needs to be corrected for higher order QCD processes such as:



Lowest order term



Order α_s QCD corrections



Inclusion of these diagrams modifies cross section prediction

Prediction now includes contributions from gluon density

Kinematics of DIS in QCD

The kinematics become more complicated than in the parton model case:

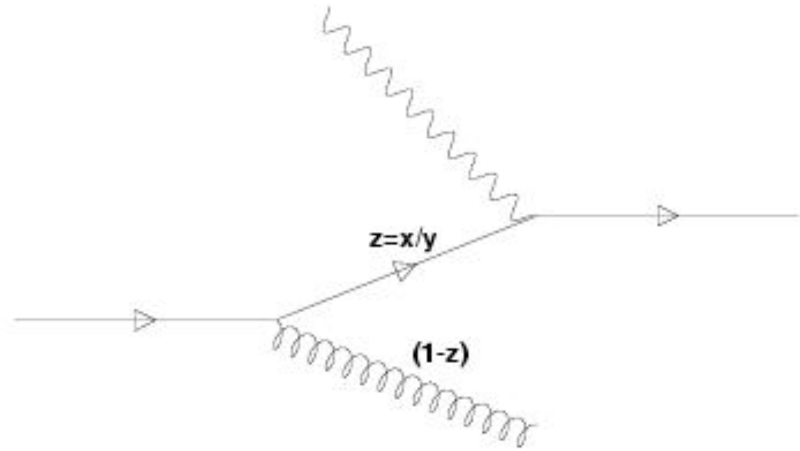
- Denote momentum fraction carried by initial quark (or gluon) y. This will be in general larger than $x=Q^2/2p\cdot q$ as measured by experiment
- This can be seen by calculating the total c.m. energy in the γ^*q frame. For the original $\gamma^*q \rightarrow q$ reaction this is just the mass of the final state parton (this will be small compared to Q^2). In the QCD-improved version it is found from the initial state particles:

$$\hat{s} = (q + yp)\cdot(q + yp)$$
$$= q^2 + y^2M_p^2 + 2yp\cdot q$$
$$= \left(\frac{y}{x} - 1\right)Q^2$$

Neglect term including proton mass

Kinematics of DIS in QCD

Let's visualise the process:



In γ^*q centre-of-mass frame energies of final state quark & gluon must be equal (denoted k′)

c.m. energy = $(2k')^2$

Energy of initial quark = k

Energy of photon = q_0 = 2k′- k (photon momentum also k)

We then have that:

$$Q^2 = k^2 - q_0^2$$
$$= 4kk' - 4k'^2$$
$$= \left(\frac{k}{k'} - 1\right)\hat{s}$$

→ Comparison of equations shows that k'/k = 1 - z

The Altarelli-Parisi equations

The additional contributions to $F_2(x,Q^2)$ can be calculated using QCD (see Halzen & Martin, chap. 10 for full details)

The cross section for the process $\gamma^* q \rightarrow qg$ is given in terms by:

$$\frac{\sigma(\gamma^* q \rightarrow qg)}{\sigma(\gamma^* q \rightarrow q)} = \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \qquad z = \frac{x}{y} = \frac{Q^2}{\hat{s} + Q^2}$$

where μ is some momentum cut-off. The splitting function

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

gives the dependence on the γ^*q c.m. energy. It can be seen as the probability of a quark emitting a gluon and so becoming a quark with energy reduced by a fraction *z*.

The Altarelli-Parisi equations

Other splitting functions can be defined:

– For finding a quark inside an initial gluon

$$P_{qg}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

- For finding a gluon inside an initial quark or gluon:

$$P_{gq}(z) = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

$$P_{gg}(z) = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right)$$

It can be seen that P_{gq} is just P_{qq} with z replaced by *(1-z),* while the two gluon splitting functions are symmetric in z and *(1-z)*

QCD corrections to the parton model

To find the DIS cross section including higher order QCD processes, assume structure functions measured at a given x include contributions from partons with higher momentum fractions y. The momentum distribution functions for different quarks flavours will then depend on Q^2:

$$f_q^{meas}(x,Q^2) = \int_x^1 dy \int_0^1 dz \left\{ f_q(y)\left(\delta(1-z) + \frac{\alpha_s}{2\pi} P_{qq}(z) \log\frac{Q^2}{\mu^2}\right) + g(y)\frac{\alpha_s}{2\pi} P_{qg}(z)\log\frac{Q^2}{\mu^2}\right\}\delta(x-yz)$$

Where we introduce the gluon momentum distribution *g(y)*.
Using the δ function to perform the z integration gives:

$$f_q^{meas}(x,Q^2) = \int_x^1 \frac{dy}{y} f_q(y)\left(\delta\left(1-\frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right)\log\frac{Q^2}{\mu^2} + g(y)\frac{\alpha_s}{2\pi} P_{qg}\left(\frac{x}{y}\right)\log\frac{Q^2}{\mu^2}\right)$$

There is a similar equation for g(x)

QCD evolution

The distribution functions now depend logarithmically on Q^2 via the dependence of the cross sections for the higher order processes. Differentiating with respect to log Q^2:

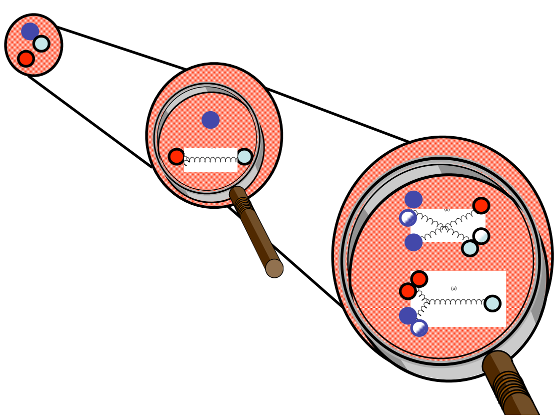
$$\frac{\partial f_q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} f_q(y,Q^2) P_{qq}\left(\frac{x}{y}\right) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2) P_{qg}\left(\frac{x}{y}\right)$$

$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} f_q(y,Q^2) P_{gq}\left(\frac{x}{y}\right) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2) P_{gg}\left(\frac{x}{y}\right)$$

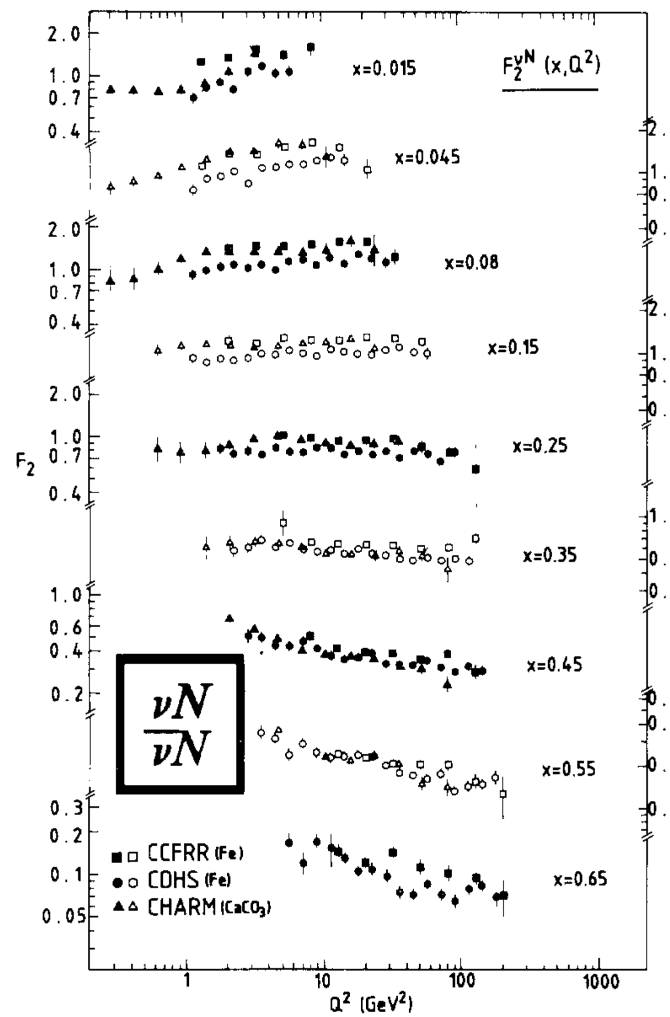
These are the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi or DGLAP evolution equations. They are extremely useful as they allow structure functions measured by one experiment to be compared to other measurements and to be extrapolated to predict what will happen in regions where no measurements exist, e.g. LHC.

Scaling violations

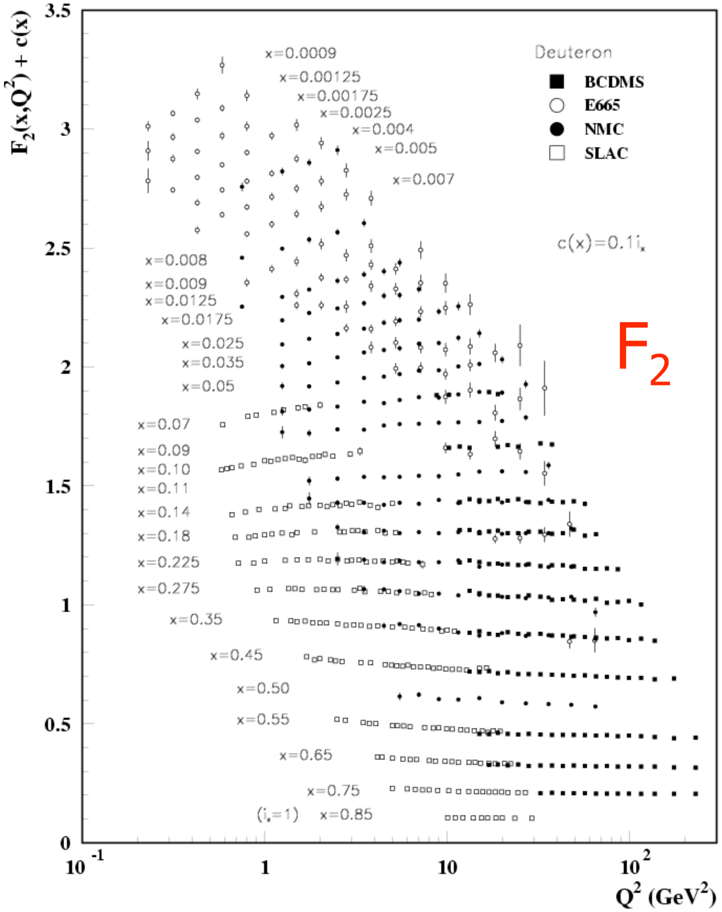
- Violations of Bjorken scaling predicted by QCD
- Logarithmic dependence on Q^2 so quite small variation
- Can be thought of as resolving more low x partons as Q^2 increases



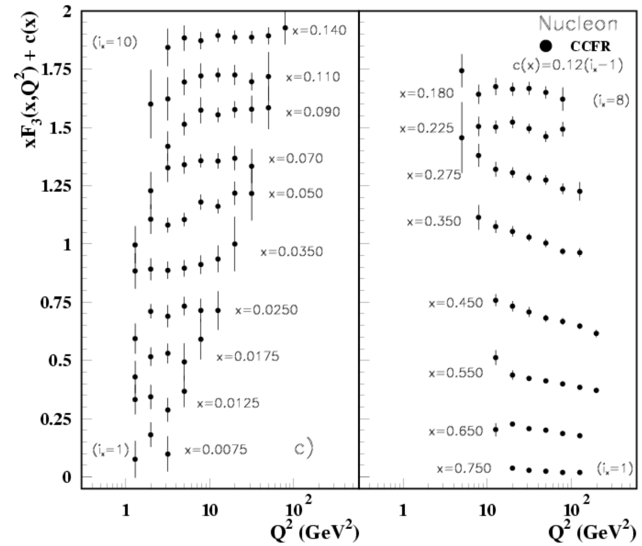
- Quantitative test of QCD



A summary of where we were



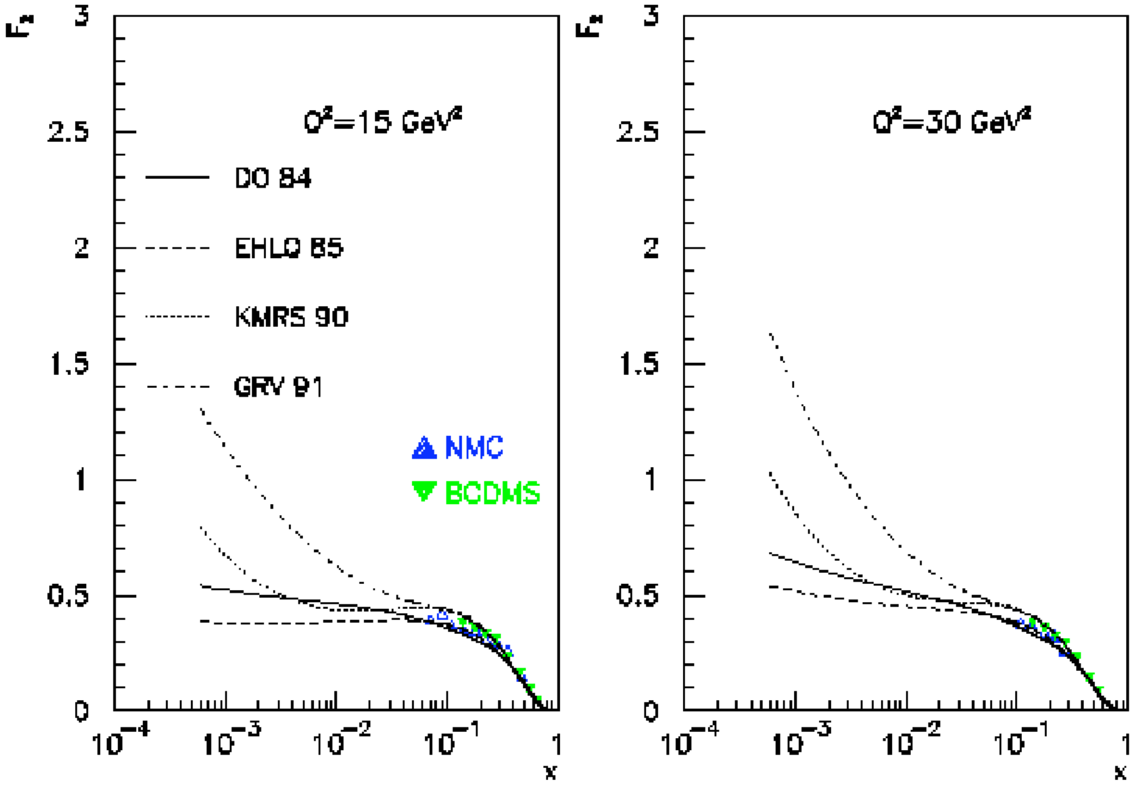
F$_2$



xF$_3$

$1\ \mathrm{GeV}^2 < Q^2 < 200\ \mathrm{GeV}^2$

Towards HERA



But what happens at low x and high Q^2?

→ Find out from HERA next time….